# Validation of Meshless Method based on Weighted Least Square Method for Simulating Electromagnetic Levitation

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**Electromagnetic levitation is a kind of magnetohydrodynamic phenomena which is used for measuring the thermo-physical properties of pure metals under high temperature. However, this phenomenon is complicated and detailed mechanisms of this phenomenon have not been clarified yet. This study proposes the meshless method based on weighted least square method for the analysis of electromagnetic levitation. In this study, the fluid motion equation and the magnetic field equation are coupled by this method. The effectiveness of this method is verified through the analysis of the molten metal behavior.**

***Index Terms*— Least squares approximations, Magnetohydrodynamics, Electromagnetic levitation, Meshless method.**

## I. Introduction

Electromagnetic levitation [1] is the method for levitating a metal by Lorentz force under high-frequency magnetic field. This method can melt active, high-purity and high-melting point metals because no container is necessary and the metal is contaminated. Therefore this technique is applied to measure the thermo-physical properties of pure metals under high temperature. However, this phenomenon is complicated and the detailed mechanisms of this phenomenon are have not been clarified yet. It is considered that numerical analysis is suited for better understanding of the phenomenon.

In magnetohydrodynamic phenomena such as the electromagnetic levitation, the fluid will be extensively deformed due to the interaction between the electromagnetic force, the surface tension and gravity. Therefore FEM is sometime not suitable for applying to such models because remeshing errors may occur due to the large deformation. Additionally, because we cannot use CAD data for deformed fluid, the material of each mesh should be defined only with the node information during the analysis. However, it is sometimes difficult to define the material of each mesh correctly if the shape of fluid is complicated. Moreover, the numerical process to generate FEM meshes does not always succeed especially when the shape of the fluid becomes complicated. In such case, the analysis will terminate.

If we are able to handle the analysis with no meshes, all difficulties mentioned above can be neglected. Recently some researchers have been studying the effectiveness of meshless method for electromagnetic problems [2]-[4]. However, the analyzed models were often very simple and limited to static field. In this paper, we propose a meshless method based on the weighted least square method [5] for the analysis of electromagnetic levitation. In this method, the fluid motion equation and the magnetic field equation are coupled by this method. The effectiveness of this method is verified by comparison with analysis results obtained by other method [6].

## II. Governing Equations

The governing equations for incompressible fluid are the Navier-Stokes equation and the mass continuity equation as follows:

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\boldsymbol{u} + \boldsymbol{g} + \frac{\boldsymbol{f_e}}{\rho} \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{2}$$

where $D/Dt$ is the Lagrangian derivative, $\boldsymbol{g}$ is acceleration of gravity, $\boldsymbol{f_e}$ is the Lorentz force density, and $\rho$, $P$, $\nu$, $\boldsymbol{u}$ are the density, pressure, kinematic viscosity, and velocity of the fluid respectively.

The surface tension $f_s$ is important when calculating fluids with free surfaces. The surface tension is shown in (3).

$$f_s = \gamma\kappa \tag{3}$$

where $\gamma$ is the surface tension coefficient, and $\kappa$ is the curvature of the fluid surface. In this paper, the surface tension calculated by (3) is not used as the external force in (1). The surface tension used as the boundary condition on the fluid surface to solve the pressure distribution inside the fluid.

On the other hand, the magnetic field equations are as follows:

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \boldsymbol{A}\right) = \boldsymbol{J_0} - \sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \nabla\varphi\right) \tag{4}$$

$$\nabla \cdot \left(\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \nabla\varphi\right)\right) = 0 \tag{5}$$

where $\mu$ is the permeability, $\boldsymbol{A}$ is the magnetic vector potential, $\boldsymbol{J_0}$ is the current density, $\sigma$ is the electric conductivity, and $\varphi$ is the electric scalar potential. The Lorentz force density $\boldsymbol{f_e}$ is given by the following equation.

$$\boldsymbol{f_e} = \boldsymbol{J_e} \times \boldsymbol{B} \tag{6}$$

where $\boldsymbol{J_e}$ is the eddy current density and $\boldsymbol{B}$ is the magnetic flux density.

In this paper, the temperature field is not considered.

## III. Analysis Method

As already mentioned, the governing equations are discretized by meshless method based on the weighted least square method in this paper. In this study, the governing equations are treated as the strong form. Therefore the discretization of the governing equations is conducted directly unlike a method with weak formulations like FEM. As a result, this method does not need the integration scheme and the calculation of the volume of calculation points.

In the fluid analysis, the fluid is discretized into groups of calculation points and the fluid flow is expressed as the motion of each calculation point. This method does not require a mesh unlike FEM. Therefore unexpected termination of an analysis due to remeshing error does not occur. In fluid analysis using the Eulerian approach, it is generally difficult to treat the advection term accurately and complex techniques are required. However, the meshless method does not need to consider the advection term in the fluid motion equation because the method employs the Lagrange approach. Moreover, the position of the free surface is directly defined by the position of the surface calculation points. Therefore the meshless method can define the free surface clearly. In this paper, the semi-implicit scheme is used to solve (1).

On the other hand, it is necessary to generate air calculation points in the air space to calculate magnetic field. In this study, air calculation points are automatically generated based on the normal lines of the calculation points on the fluid surface.

After the generation of air calculation points, the distribution of the magnetic field and the Lorentz force at each fluid calculation point are calculated by the weighted least square method. Finally, the fluid motion which is affected by the Lorentz force is calculated by the weighted least square method and those process repeats till end time.

## IV. Analyzed Results and Conclusion

The analyzed model in initial state is shown in Fig. 1. The induction coil is simplified into 4 ring coils in different diameter (5 mm, 7.5 mm, 10 mm and 12.5 mm). A spherical metal is placed upward the induction coil. In this paper, it is assumed that the metal is already molten in this initial state and the phase transformation of the metal is not considered. In addition, the shape of molten metal is assumed to be a cylinder with infinite depth because this is a 2D analysis in this short paper.

The calculated distribution of the Lorentz force when alternating current (300 A, 10000Hz) was applied to the induction coil is shown in Fig. 2. The Lorentz force acts as a levitating force against gravity. The distribution of calculation points during the analysis is shown in Fig. 3. In Fig. 3, the blue, red and green points represent the fluid, coil, and air calculation points respectively. The air calculation points are automatically generated based on the shape of the molten metal. At the same time, the molten metal is levitated by the Lorentz force and fluctuates due to the interdependence of the Lorentz force and the surface tension. Through the analysis, it is conjectured that the proposed analysis method can be an effective tool for calculating magnetohydrodynamic problems with large deformation. In the extended version of this paper, 3D analysis results and a validation of this method will be shown.

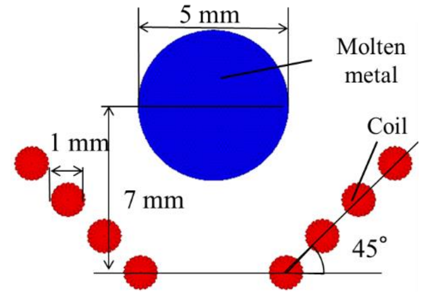
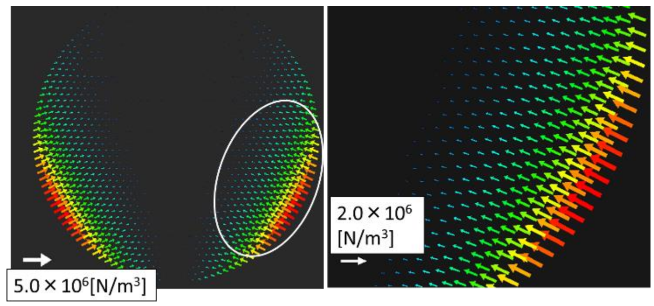
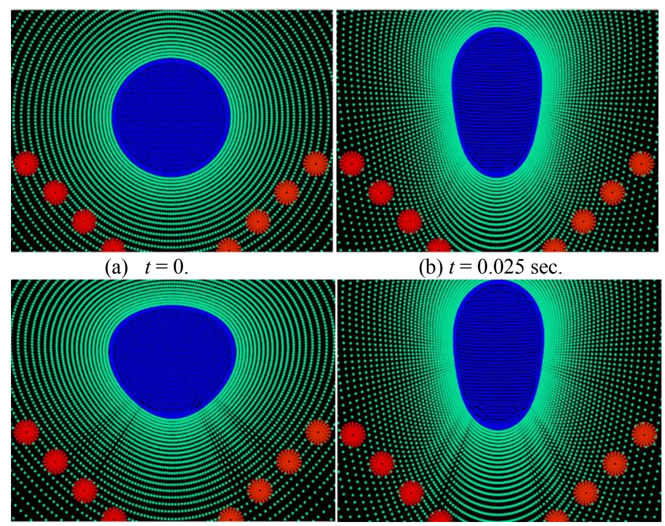



Fig. 1. Analyzed model of initial state.




(a) Overall view. (b) Enlarged view inside the white circle.

Fig. 2. Distribution of the Lorentz force inside molten metal at initial state.



(a) $t = 0$. (b) $t = 0.025$ sec.

(c) $t = 0.035$ sec. (d) $t = 0.045$ sec.

Fig. 3. Distribution of the calculation point around the molten metal.

## V. References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